For all questions, answer (E) NOTA means none of the above answers is correct. All numbers on this test are real numbers. All functions on this test have domains and ranges that are subsets of the real numbers.

1. Let $f(x) = \begin{cases} x, & x \text{ is rational number} \\ 0, & x \text{ is irrational number} \end{cases}$. Evaluate $\lim\limits_{x \to 0} f(x)$.

    (A) 0   (B) 1   (C) -1   (D) not exists   (E) NOTA

2. Let $f(x) = \begin{cases} 0, & x \neq 0 \\ 1, & x = 0 \end{cases}$.   Evaluate $\lim\limits_{x \to 0} f(f(x))$.

    (A) 0   (B) 1   (C) -1   (D) not exists   (E) NOTA

3. Let $f(x) = x^2 + 1$.   Evaluate $\lim\limits_{x \to 2} f(x)$.

    (A) 1   (B) 2   (C) 5   (D) 0   (E) NOTA

4. Let $f(x) = \dfrac{x-1}{x^2+1}$.   Evaluate $\lim\limits_{x \to 1} f(x)$.

    (A) 1   (B) 2   (C) ∞   (D) 0   (E) NOTA

5. Let $f(x) = \dfrac{\sqrt{4x^3 + 2x^2 - 5x}}{\sqrt{x^3 - x} - \sqrt{4x}}$.   Evaluate $\lim\limits_{x \to \infty} f(x)$.

    (A) 1   (B) 2   (C) 5   (D) 4   (E) NOTA

6. Let $f(x) = \dfrac{x^4 - 2x - 12}{x^5 - 2x^2 - 24}$.   Evaluate $\lim\limits_{x \to 2} f(x)$.

    (A) 0   (B) 2   (C) $\dfrac{4}{5}$ (D) $\dfrac{5}{12}$ (E) NOTA

7. Let $f(x) = \dfrac{2\sin x - \sin 2x}{x - \sin x}$.   Evaluate $\lim\limits_{x \to 0} f(x)$.

    (A) 2   (B) -4   (C) 6   (D) -8   (E) NOTA

8. Let $f(x) = \dfrac{3^{2x} - 1}{2x}$.   Evaluate $\lim\limits_{x \to 0} f(x)$.

    (A) 0   (B) 1   (C) $\ln 2$ (D) $\ln 3$ (E)
NOTA

9. Let $f(x) = \dfrac{\ln(4\sin x + 1)}{2x}$.   Evaluate $\lim\limits_{x \to 0} f(x)$.

    (A) 0   (B) 1   (C) 2   (D) 3   (E) NOTA

10.   Let $f(x) = \dfrac{3}{1 - 3^{-\frac{1}{x}}}$.   Evaluate $\lim\limits_{x \to 0^+} f(x)$.

(A) 0   (B) 1   (C) 2   (D) 3   (E) NOTA

11.   Let $f(x) = x \sin \dfrac{1}{x^2}$.   Evaluate $\lim\limits_{x \to 0} f(x)$.

(A) 0   (B) 1   (C) 2   (D) 3   (E) NOTA

12.   Let $f(x, y) = \dfrac{2x^2 y}{x^2 + y^2}$.   Evaluate $\lim\limits_{(x,y) \to (0,0)} f(x, y)$.

(A) 0   (B) 1   (C) 2   (D) 3   (E) NOTA

13.   Let $f(x) = x^3 + x - 1$.   Which of the following is not a possible value of $f'(x)$?

(A) 0   (B) 1   (C) 2   (D) 4   (E) NOTA

14.   Let $f(x) = \begin{cases} x^2, x \geq 2 \\ x + 2, x < 2 \end{cases}$. Suppose $\lim\limits_{x \to a} f(x) = f'(a)$, Then what is the value of a?

(A) 0   (B) 1   (C) -1   (D) 2   (E) NOTA

15.   Let $f(x) = \begin{cases} x + 1, x \neq 1 \\ 3, x = 1 \end{cases}$. Evaluate $\lim\limits_{x \to 1} f(x)$

(A) 0   (B) 1   (C) 2   (D) 3   (E) NOTA

16.   Evaluate $\lim\limits_{x \to 1} \sin^{-1}\left(\dfrac{1 - \sqrt{x}}{1 - x} \right)$

(A) 0   (B) $\dfrac{\pi}{6}$ (C) $\dfrac{\pi}{3}$ (D) $\dfrac{\pi}{2}$ (E) NOTA

17.   Let $f(x) = x^4 + \sin x^2 + e^{\cos x} + 5$. If $g(x) = f(x) + (f'(x))^2 + f''(x)$, evaluate $g(0)$.

(A) 1   (B) 2   (C) 4   (D) 7   (E) NOTA

18.   Let $f(x) = \int_0^{x^2} (\sin t^2 + e^{\sin^{-1} t}) dt$. Evaluate $f''(0)$

(A) 0   (B) 1   (C) 2   (D) 4   (E) NOTA

19. Evaluate $\lim\limits_{n\to\infty}\sum\limits_{i=2}^{n}\dfrac{4}{3(i^2-1)}$

(A) $\dfrac{1}{3}$ (B) $\dfrac{2}{3}$ (C) 1   (D) $\dfrac{4}{3}$ (E) NOTA

20. Let $f(x)=\dfrac{12(x-1)^{\frac{2}{3}}\sqrt{x+1}}{(1-x^2)^3}$. Evaluate $f'(0)$.

(A) 2   (B) 1   (C) -1   (D) -2   (E) NOTA

21. Suppose $x^3+y^2=80$, evaluate $(\dfrac{dy}{dx}-\dfrac{d^2y}{dx^2})|_{(4,4)}$

(A) 0   (B) 1   (C) -1 (D) 2   (E) NOTA

22. Let $f(x)=\cos\dfrac{a}{x}$ where $a\neq0$ is a constant. Evaluate $\lim\limits_{x\to\infty}f(x)$.

(A) 0   (B) $-a$ (C) 1   (D) a (E) NOTA

23. If $y^5\sin x^2-\cos(x-y)=0$, evaluate $\dfrac{dy}{dx}\Big|_{(0,\frac{\pi}{2})}$.

(A) 0   (B) -1 (C) 1   (D) 5 (E) NOTA

24. Evaluate $\lim\limits_{x\to0}\dfrac{\cos^{2017}x-1}{\cos x-1}$

(A) 0   (B) 2017   (C) -2017   (D) $2017!$ (E) NOTA

25. Evaluate $\lim\limits_{x\to\frac{\pi}{2}}(1+\cot x)^{\frac{1}{\cos x}}$

(A) e   (B) e^{-1} (C) -e   (D) e^2 (E) NOTA

26. Let $y=\sqrt{x+a\sqrt{x+a\sqrt{x+a\sqrt{x+\cdots}}}}$ where $a\neq0$ is a constant. Evaluate $\dfrac{dy}{dx}$.

(A) $\dfrac{1}{2y+a}$ (B) $\dfrac{1}{a-2y}$ (C) $\dfrac{1}{2y-a}$ (D) $\dfrac{a^2}{2y+a}$ (E) NOTA

27.   Evaluate $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{i+n}$

(A) 1   (B) $\ln 2$ (C) $\ln 3$ (D) $\ln 5$ (E) NOTA

28.   Evaluate $\displaystyle\lim_{x\to\pi}\sin(x+\sin x)$

(A) 0   (B) 1   (C) 2   (D) 3   (E) NOTA

29.   Find an equation of the tangent line to the curve $f(x)=2x\sin x$ at point $x=\dfrac{\pi}{2}$.

(A) $y=\pi x$ (B) $y=x$ (C) $y=2x$ (D) $y=\dfrac{\pi}{2}x$ (E) NOTA

30.   Let $f(x)=x\sin^{-1}x+\sqrt{1-x^2}$ Evaluate $f'(\dfrac{1}{2})$.

(A) 0   (B) $\dfrac{\pi}{6}$ (C) $\dfrac{\pi}{3}$ (D) $\dfrac{\pi}{2}$ (E) NOTA